Skupovi

Skup je grupa objekata predstavljenih kao cjelina. Skup može sadržavati bilo koji tip objekata uključujući brojeve, simbole ili čak neke druge skupove. Objekti u skupu se zovu elementi ili članovi. Skupovi mogu biti opisani na više načina

$\{7, 21, 57\}$

$\{n \mid n=2m \wedge m \in \mathbb{N}\}$

$\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$

Simbolima $\in$ i $\notin$ označavamo da li je neki element član ili nije član skupa.

$A = B$ znači da je za svaki x, $x \in A$ akko $x \in B$

$$A = B \Leftrightarrow \forall(x)(x \in A \Leftrightarrow x \in B)$$

npr. $\mathbb{N}_0 = \{0, 1, 2, \dots\}$

Skup može biti konačan (npr. $\{7, 4, 33\}$) ili beskonačan (npr. $\mathbb{N}$). Pitanje: Da li je skup $\{\mathbb{N}\}$ konačan?

A je podskup od B (pišemo $A \subseteq B$) ako je svaki član od A također član od B.

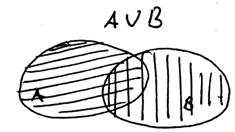
$$A \subseteq B \Leftrightarrow \forall(x)(x \in A \Rightarrow x \in B).$$

Prazan skup označavamo sa $\emptyset$

Operacije na skupovima

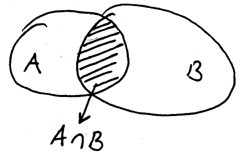
$\cup$ unija, npr. $\{a,b\} \cup \{b,c\} = \{a,b,c\}$

$A \cup B = \{x \mid x \in A \vee x \in B\}$



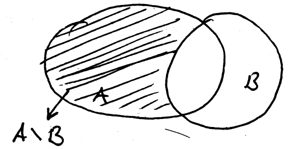
$\cap$ presjek, npr. $\{a,b\} \cap \{b,c\} = \{b\}$

$A \cap B = \{x \mid x \in A \wedge x \in B\}$



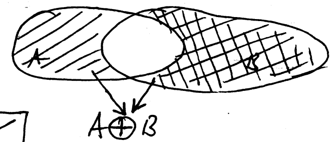
$\setminus$ razlika, npr. $\{a,b\} \setminus \{b,c\} = \{a\}$

$A \setminus B = \{x \mid x \in A \wedge x \notin B\}$



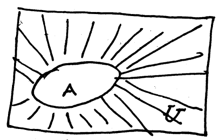
$\oplus$ simetrična razlika ili disjunktna suma, npr. $\{a,b,c\} \oplus \{c,d,e\} = \{a,b,d,e\}$

$A \oplus B = \{x \mid x \in A \setminus B \vee x \in B \setminus A\}$



C komplement (dopuna),

$C(A) = \{x \mid x \notin A\}$



npr. $\{a,b,c\}$ $\quad C(\{a,b,c\}) = \{e,f,g\}$

ako je $U = \{a,b,c,e,f,g\}$

× Dekartov proizvod,

$A \times B = \{(a,b) \mid a \in A \wedge b \in B\}$

(a,b) čitamo: par brojeva a i b

npr. $\{a,b\} \times \{a\} = \{(a,a),(b,a)\}$

$\mathcal{P}$ partitivni skup,

$\mathcal{P}(A) = \{S \mid S \subseteq A\}$

npr. $\mathcal{P}(\{a,b\}) = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$

Pitanje: Koliko elemenata ima skup $\mathcal{P}(\{a,b\})$.

1. Dati su skupovi $A = \{x \mid x \in \mathbb{N} \wedge -2 \leq x < 4\}$,
$B = \{x \mid x \in \mathbb{Z} \wedge -1 \leq x < 3\}$ i $U = \{x \mid x \in \mathbb{Z} \wedge -3 \leq x \leq 5\}$

Nađi: $C(A \cap B)$, $C(A) \setminus C(A \cup B)$ i $C(A \oplus B) \cap C(B)$.

Rj. Skupovi A, B i C su konačni skupovi

$A = \{1,2,3\}$

$B = \{-1,0,1,2\}$

$U = \{-3,-2,-1,0,1,2,3,4,5\}$

$A \cup B = \{x \mid x \in A \vee x \in B\} = \{-1,0,1,2,3\} = \{x \mid x \in \mathbb{Z} \wedge -1 \leq x \leq 3\}$

$A \cap B = \{x \mid x \in A \wedge x \in B\} = \{1,2\} = \{x \mid x \in \mathbb{N} \wedge 1 \leq x < 3\}$

$C(A) = \{x \mid x \notin A\} = \{-3,-2,-1,0,4,5\} = U \setminus A$

$C(B) = \{x \mid x \notin B\} = \{-3,-2,3,4,5\}$

$C(A \cap B) = \{x \mid x \notin A \cap B\} = \{-3,-2,-1,0,3,4,5\}$ ←

$C(A \cup B) = \{x \mid x \notin A \cup B\} = \{-3,-2,4,5\}$

$C(A) \setminus C(A \cup B) = \{x \mid x \in C(A) \wedge x \notin C(A \cup B)\} = \{-1,0\}$ ←

$A \oplus B = \{x \mid x \in A \setminus B \vee x \in B \setminus A\} = \{-1,0,3\}$

$A \setminus B = \{x \mid x \in A \wedge x \notin B\} = \{3\}$

$B \setminus A = \{x \mid x \in B \wedge x \notin A\} = \{-1,0\}$

$C(A \oplus B) = \{-3,-2,1,2,4,5\}$

$C(B) = \{-3,-2,3,4,5\}$

$C(A \oplus B) \setminus C(B) = \{x \mid x \in C(A \oplus B) \wedge x \notin C(B)\} = \{1,2\}$ ←

2. Dokazati da za proizvoljne skupove A, B i C važi
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Rj. uzmimo proizvoljan $x \in A \cap (B \cup C)$

$x \in A \cap (B \cup C) \Rightarrow x \in A \wedge x \in B \cup C \Rightarrow x \in A \wedge (x \in B \vee x \in C)$

$\Rightarrow (x \in A \wedge x \in B) \vee (x \in A \wedge x \in C) \Rightarrow x \in A \cap B \vee x \in A \cap C$

$\Rightarrow x \in (A \cap B) \cup x \in (A \cap C)$

čime smo dokazali da je $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$ (*)

uzmimo proizvoljan $x \in (A \cap B) \cup (A \cap C)$

$x \in (A \cap B) \cup (A \cap C) \Rightarrow x \in A \cap B \vee x \in A \cap C \Rightarrow (x \in A \wedge x \in B) \vee$

$\vee (x \in A \wedge x \in C) \Rightarrow x \in A \vee (x \in B \wedge x \in C) \Rightarrow$

$\Rightarrow x \in A \vee x \in B \cap C \Rightarrow x \in A \cup (B \cap C)$

čime smo dokazali da je $(A \cap B) \cup (A \cap C) \subseteq A \cup (B \cap C)$ (**)

(*) $\wedge$ (**) $\Rightarrow$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

q.e.d.

3. Dati su skupovi $A = \{x \in \mathbb{N} \mid x = 2n,\ n \in \mathbb{N}\}$ i
$B = \{x \in \mathbb{N} \mid x = 3n,\ n \in \mathbb{N}\}$. Nađi skup $A \cap B$.

Rj. A i B su beskonačni skupovi. Napišimo elemente skupova A i B.

$A = \{2,4,6,8,10,12,14,16,18,\dots\}$

$B = \{3,6,9,12,15,18,\dots\}$

Vidimo da $A \cap B$ postoji, jer npr. 6 je element i skupa A i skupa B.

Uzmimo proizvoljan $x \in A \cap B$.

$x \in A \cap B \Rightarrow x \in A \land x \in B \Rightarrow$

$\Rightarrow \exists(n \in \mathbb{N})\ x = 2n \land \exists(m \in \mathbb{N})\ x = 3m \Rightarrow$

$\Rightarrow$ x je djeljiv sa 2 $\land$ x je djeljiv sa 3 $\Rightarrow$

$\Rightarrow$ x je djeljiv sa 6 $\Rightarrow \exists(k \in \mathbb{N})\ x = 6k$

odavdje zaključujemo: $A \cap B = \{x \in \mathbb{N} \mid x = 6k,\ k \in \mathbb{N}\}$

4. Prikazati u koordinatnom sistemu $A \times B$ ako je

$A = \{x \in \mathbb{R} \mid 1 \le x \le 3\}$

$B = \{x \in \mathbb{R} \mid 1 \le x \le 2 \lor 3 \le x \le 5\}$.

Rj. Prije nego što uradimo zadatak posmatrajmo skupove C i D koji su podskupovi od A i B tako da je $x \in \mathbb{N}$ tj.

$C = \{x \in \mathbb{N} \mid 1 \le x \le 3\}$

$D = \{x \in \mathbb{N} \mid 1 \le x \le 2 \lor 3 \le x \le 5\}$

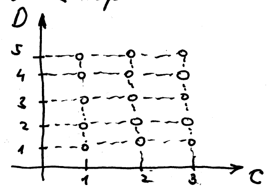
Skupovi C i D su konačni

$C = \{1, 2, 3\}$

$D = \{1, 2, 3, 4, 5\}$

$C \times D = \{(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (2,5),$
$(3,1), (3,2), (3,3), (3,4), (3,5)\}$

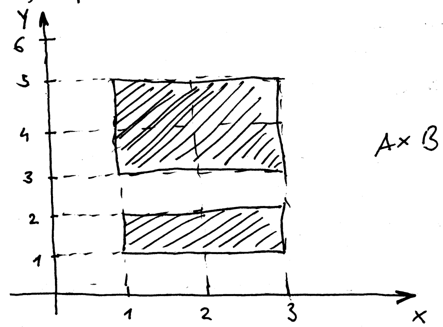
Predstavimo pro skup $C \times D$ u koordinatnom sistemu:



tačke o predstavljaju elemente skupa $C \times D$.

$A \times B = \{(a,b) \mid a \in A \land b \in B\} = \{(a,b) \mid 1 \le a \le 3 \land (1 \le b \le 2 \lor 3 \le b \le 5)\}$

$= \{(x,y) \mid 1 \le x \le 3 \land (1 \le y \le 2 \lor 3 \le y \le 5) \land x \in \mathbb{R} \land y \in \mathbb{R}\}$



$A \times B$